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THE PENNSYLVANIA
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IONOSPHERIC RESEARCH

Scientific Report No. 157

THE INTERPLANETARY MAGNETIC FIELD AND THE AURORAL ZONES

by

J. W. Dungey

March 15, 1962

The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Laboratory, Air Research and Development Command, under Contract AF19(604)-4563 and, in part, by the National Science Foundation under Grant G-18983.

IONOSPHERE RESEARCH LABORATORY

University Park, Pennsylvania

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Approved for Distribution

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Abstract

It has been found that a model with a southward interplanetary magnetic field leads to a natural explanation of the SD currents and a short account of this has been published (Dungey 1961). This report gives more detail and discusses some more speculative aspects of the problem as they appear at this time. It should be remembered that this problem is amenable to revolutionary progress by observations from rockets or satellites which go out more than a few earth's radii.

1. Existence of Neutral Points

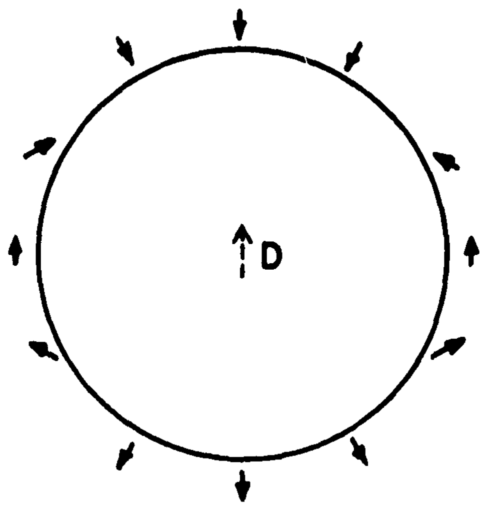
The most important feature in this problem is the existence of neutral points of the magnetic field; their existence is very plausible but so important as to perhaps merit some attempt at proof. It may be noted that two neutral points do occur on the surface of the Chapman-Ferraro cavity in the absence of an interplanetary field. This is in the idealized model, where the screening current is taken to flow in a mathematical surface, and, more important, the magnetic field outside the cavity is taken to vanish. It would be more strictly accurate to allow that a minute quantity of magnetic flux leaks out and then there is a finite field outside the current layer; even though it decreases exponentially with a tiny scale as one travels away from the earth. Strictly then there are not neutral points without an interplanetary field, but there so nearly are, that the slightest interplanetary field will make them.

The question of the existence of neutral points can be tackled by taking the situation in which there is an interplanetary field but no interplanetary wind and then trying to understand the change that takes place when the wind starts to blow. In the case of a uniform field superposed on a dipole field the existence of two neutral points can be proved quite simply. If the uniform field is exactly south, there is a neutral circle in the equatorial plane and, if it is due north, there are two neutral points on the dipole axis. Otherwise the dipole and the direction of the uniform

field define a plane, to which attention is now confined.

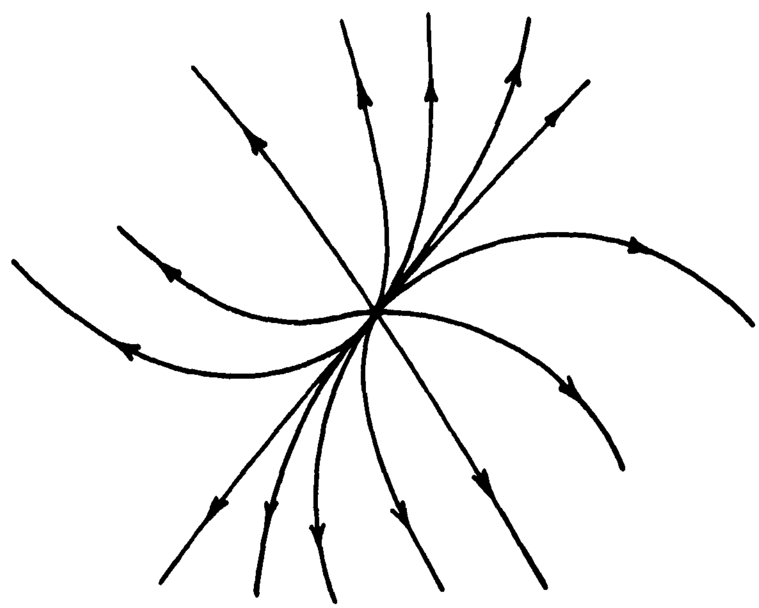
Fig. 1 shows the direction of the dipole field alone on a circle centered on the dipole. As one travels round this circle the direction of the dipole field rotates and makes two complete revolutions for one circuit of the circle. There are, therefore, two points on the circle at which the direction of the dipole field alone is opposite to that of the uniform field. Now, if one travels out from the dipole on a radial straight line, the dipole field has the properties that its direction is the same all along this line, while its strength varies as r^{-3}. Consequently the dipole field has the opposite direction to that of the uniform field all along a straight line through the dipole, and there are two points, one on each side of the dipole, where the strength of the dipole field equals that of the uniform field. On superposing the fields these two points become neutral points, which are situated symmetrically with respect to the dipole. For an interplanetary field of 4 γ, they would be about 20 earth radii from the earth.

The development of the field after a wind starts to blow is much more complicated, but we restrict attention to the question whether the neutral points can disappear. Any change must take place continuously, so that one must consider first an infintesimal change in the magnetic field. Then there are still two neutral points, but in slightly different positions. Thus the wind will move the neutral points. They could be removed by blowing them away to infinity,



DIRECTION OF A MAGNETIC FIELD ON A CIRCLE SURROUNDING A DIPOLE "D"

FIGURE 1



DIRECTION OF THE MAGNETIC FIELD IN THE $\vec{e}_1$, $\vec{e}_2$ PLANE AROUND A NEUTRAL POINT

FIGURE 2

but this is clearly equivalent to reverting to the case with zero interplanetary field, and could not happen, if the incoming plasma contained a field. It is possible that the neutral points could be destroyed, if they were blown together. This cannot be discussed without further knowledge of the topology of the field.

2. Topology of the Field

Assuming that neutral points exist, the topology of the field is the same as that for a dipole field with a uniform field superposed. The topology is important and its understanding requires consideration of the field in the neighborhood of a neutral point. Because the field must be a solution of Maxwell's equations, it must be differentiable, and hence expansible in a Taylor series. It is sufficient to stop after the first order terms and then in the neighborhood of a neutral point the field is described by the second order tensor $\partial B_i/\partial x_j$.

Consider the principal axes of this tensor. Taking the neutral point as origin, we seek a neighboring point with position vector $\underline{\delta x}$ where the field $\underset{\sim}{B}$ is parallel (or antiparallel) to $\underline{\delta x}$. In the limit $\delta x \to 0$, this is expressed by

$$B_i = \frac{\partial B_i}{\partial x_j} \delta x_j = \lambda \delta x_i \tag{1}$$

where λ is any scalar. Equation (1) determines certain directions for $\underline{\delta x}$ which, by definition, are the directions of the principal axis of $\partial B_i/\partial x_j$. Thinking of $\partial B_i/\partial x_j$ as a

matrix, Eqn. (1) is the equation for the eigenvalues λ and eigenvectors $\underline{\delta x}$. In three dimensions two of the three eigenvalues and the corresponding eigenvectors can be complex. If so, however, the neutral point is O type (elliptical), whereas those in the dipole plus uniform field are X type (hyperbolic), so thatwe need onlv the case where all three values are real. The Maxwell equation div $\underset{\sim}{B} = 0$ puts a constraint on the tensor $\partial B_i/\partial x_j$, which in matrix terms is the vanishing of the trace. Now a matrix theorem states that the sum of the eigenvalues equals the trace. Then, since the sum of the eigenvalues vanishes, the eigenvalues cannot all have the same sign.

At this point Sydney Chapman mentions the Cambridge hostess who offered accomodation for three people provided at least two were of the same sex. Likewise two of three real eigenvalues must have the same sign. Let the eigenvectors be $\underset{\sim}{e}_1$, $\underset{\sim}{e}_2$, $\underset{\sim}{e}_3$ (unit vectors) and the corresponding eigenvalues be μ_1, μ_2, $-\mu_3$ where the μ's are positive. Then, when ε_1, ε_2 and ε_3 are infinitesimal, the field at position $\underset{\sim}{r} = \varepsilon_1\underset{\sim}{e}_1 + \varepsilon_2\underset{\sim}{e}_2 + \varepsilon_3\underset{\sim}{e}_3$ is

$$\varepsilon_1\mu_1\underset{\sim}{e}_1 + \varepsilon_2\mu_2\underset{\sim}{e}_2 - \varepsilon_3\mu_3\underset{\sim}{e}_3 \tag{2}$$

Now take the plane $\varepsilon_3 = 0$, that is the plane containing the principal axes with directions $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$. Near the neutral point in this plane

$$\underset{\sim}{B} \cdot \underset{\sim}{r} = \varepsilon_1^2\mu_1 + \varepsilon_2^2\mu_2 \tag{3}$$

which shows that the radial component of the field is outwards all round the neutral point. Typical behavior of the lines of force is shown in Fig. 2.

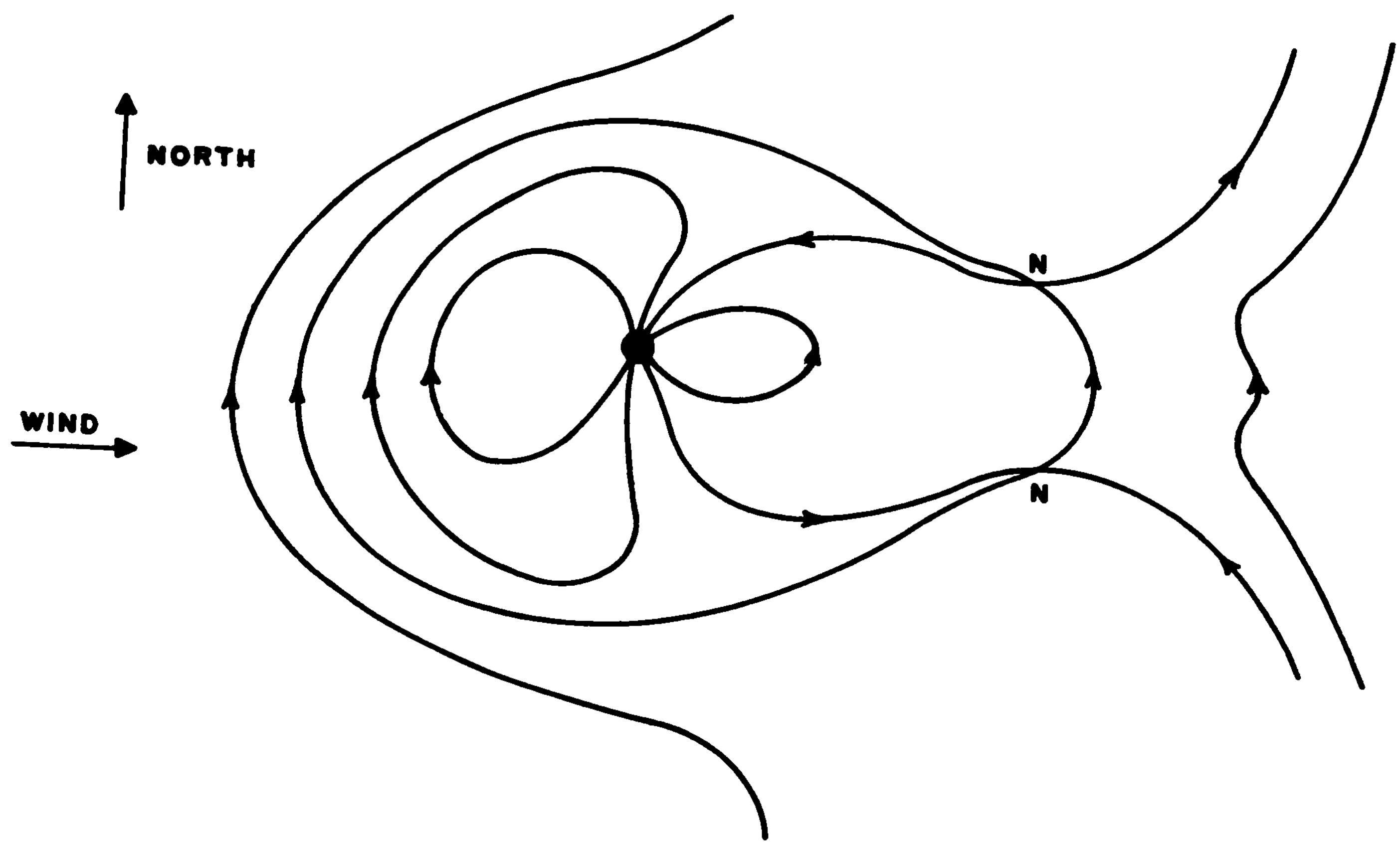
In the case illustrated all the lines of force in the plane $\varepsilon_3 = 0$ (Fig. 2) come out from the neutral point and these lines are infinite in number and must cover some surface. On the other hand, only two lines go in to the neutral point, namely those in directions $\pm e_3$. Of one considers a thin tube of force enclosing one of these lines, this tube "splays out" as it approaches the neutral point and its continuation is a surface close to that covered by the lines coming out of the neutral point. Thus the neutral point has an essential asymmetry in that one incoming line or very thin tube gets spread out to cover a whole surface. Of course the opposite asymmetry is equally possible. If two of the eigenvalues μ_1, μ_2, μ_3 are negative, there is an infinity of lines going in and only two coming out. Now, since lines of force must be endless, it is impossible to have just one neutral point of this type, because if one follows the infinity of lines far enough one must get back to the two lines.

It was seen in Section 1 that we are interested in a field with two neutral points, and it is now seen that their asymmetries must have opposite senses, so that the excess of outgoing lines of force from one is absorbed by the second. In the simplest cases the surface of outgoing lines from one is actually the same as the surface of lines going

in to the other. In our problem this can be true, if suitable continuations through the earth and through distant space are assumed.

The model to be discussed has a nearly southward interplanetary magnetic field, but a nearly northward magnetic field is favored by H. Elliot to fit cosmic ray observations and fits better the results from Explorer X, so this case will be outlined first. If a northward uniform field is superposed on the dipole field, the neutral points are near the lines of force from the poles. The effect of the wind is expected to blow the neutral points further over to leeward as in Fig. 3. As a point just in front of the paper the sign of the component of the field perpendicular to the paper is the same as for the dipole alone. Thus just in front of the northern neutral point, the field points in to the neutral point and vice versa. Thus there is a surface covered by lines of force which enter the northern neutral point and here the surface is roughly parallel to the equatorial plane. Two lines in this surface appear in Fig. 3; they both go directly from the southern neutral point to the northern neutral point. In the absence of a wind and with the uniform field due north the field is symmetrical about the geomagnetic axis, so that the surface is the surface of revolution generated by either of the lines mentioned in Fig. 3.

The topology is slightly changed by a slight tilt of the interplanetary field as will be discussed later, but should not be changed by a wind. The topology for a due



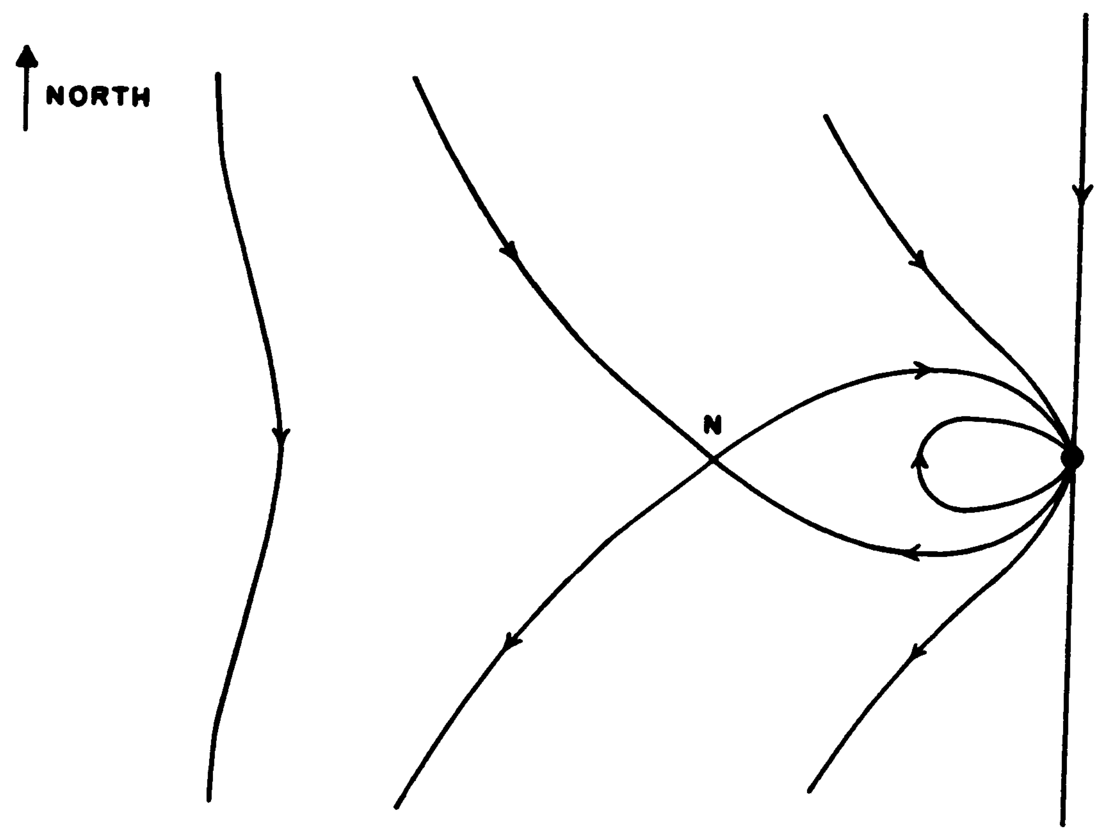
MAGNETIC FIELD TOPOLOGY IN MERIDIAN PLANE FOR A NORTHWARD INTERPLANETARY FIELD AND A WIND. NEUTRAL POINTS OCCUR AT "N".

FIGURE 3

north field then is such that the lines coming out of the southern neutral point all go directly to the northern neutral point and cover a surface which encloses the earth (with reasonable numerical values the radius of this surface is at least 10 earth radii).

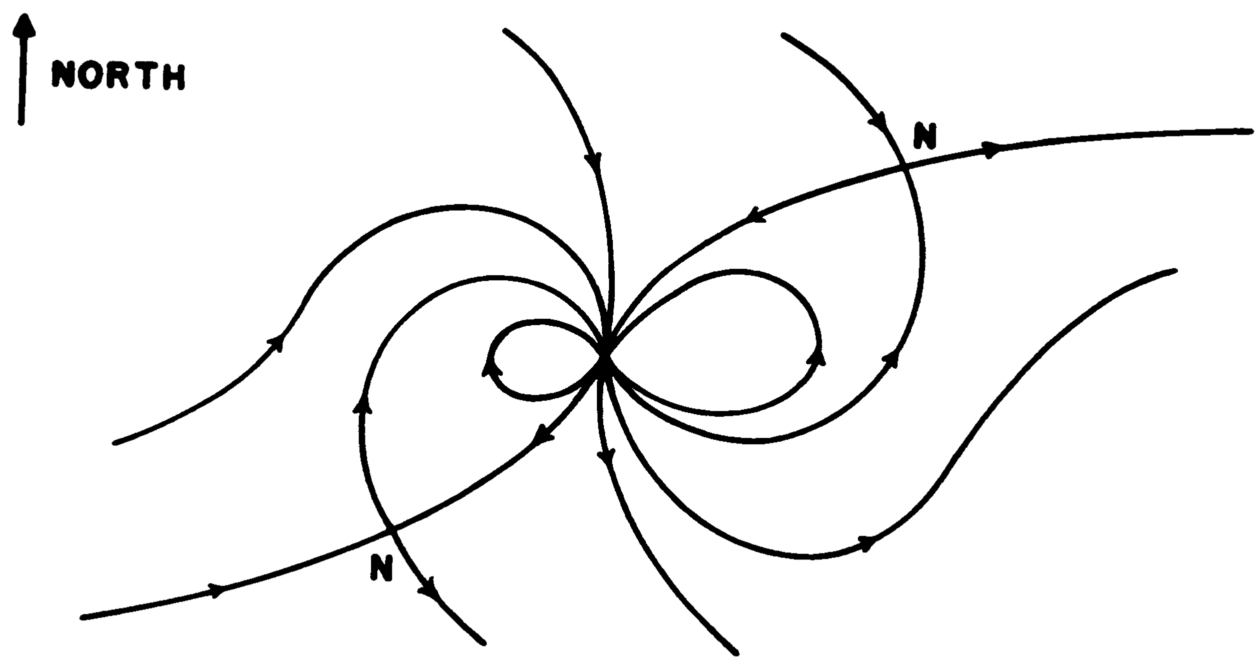
The topology for a southward interplanetary field is more complicated. For a due south field and no wind, there is axial symmetry, and the field in any meridian plane is shown in Fig. 4. In this singular case there is a neutral ring in the equatorial plane. A wind would destroy the axial symmetry, but there would still be planes of symmetry and still a neutral ring. If the interplanetary field is tilted, however little, there are just two neutral points, which are on opposite sides of the equatorial plane, as in Fig. 5. The field at a point just in front of the paper is obtained as before and is inward just in front of the neutral point in the northern hemisphere. There is therefore a surface of lines going into the northern neutral point, and this includes two of the lines shown in Fig. 5, one coming from near the south pole of the earth and one coming from northern space. Lines coming from the southern neutral point cover a surface, which is the other way round.

Consider the lines going into the northern neutral point. Some come from the southern polar region and some from northern space and there must be a point where these lines separate. This point must be an X type neutral point and therefore can only be the southern neutral point. Thus



MAGNETIC FIELD TOPOLOGY IN MERIDIAN HALF PLANE FOR A SOUTHWARD INTERPLANETARY FIELD AND NO WIND

FIGURE 4



MAGNETIC FIELD TOPOLOGY IN MERIDIAN PLANE FOR SOUTHWARD, "TILTED," INTERPLANETARY FIELD

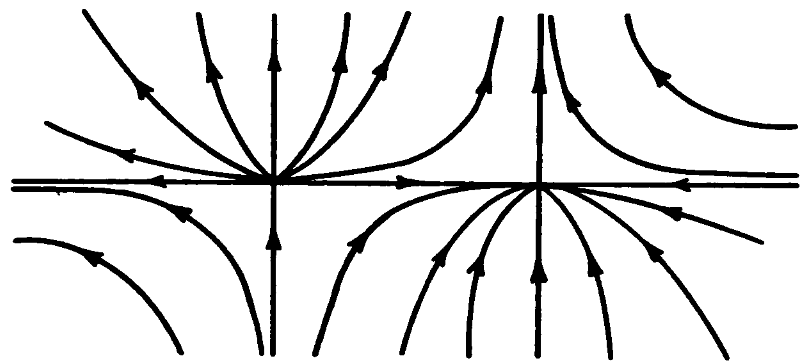
FIGURE 5

there is a line which goes directly from the southern to the northern neutral point; in fact there are two, one in front of the paper in Fig. 5 and one behind. These two lines form a closed curve and the surfaces belonging to the two neutral points cross on this loop. Together these surfaces form two surfaces which are topologically like a cylinder going off into space and a doughnut which intersects the earth. It is easily seen from Fig. 5 that these surfaces separate lines which have two, one or no feet on the ground. Fig. 6 shows the way the lines of force are arranged on either the cylinder or the doughnut, in this diagram the surface has been "unwrapped" and flattened, so that in reality the sides are joined.

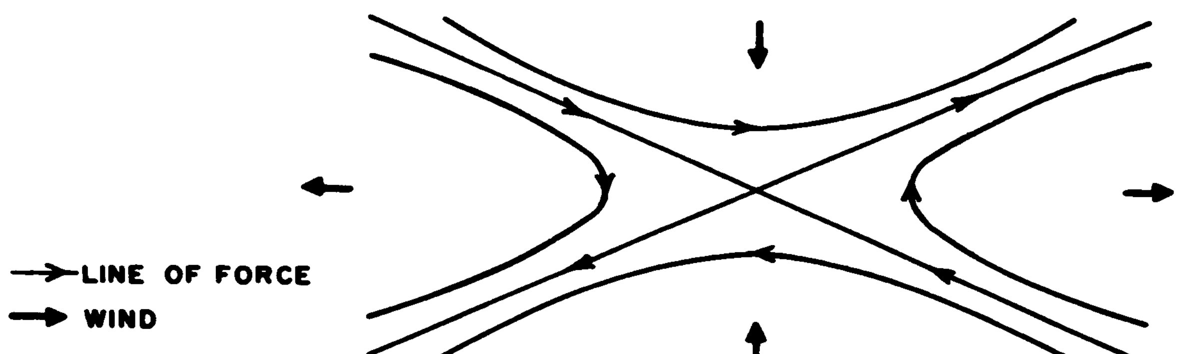
3. Motion of the Plasma

The motion of the plasma in the region where it interacts with the earth's field involves all the worst difficulties possible in plasma physics. It is certainly not dominated by collisions and the flow is quite complicated enough to set up important anisotropy of the velocity distribution of the protons. Anisotropy almost certainly leads to microinstabilities, which have the effect of reducing the degree of anisotropy, but the theory of these processes is still in its infancy. Consequently there must be great uncertainty about some aspects of our problem, but fortunately a general outline can be obtained by neglecting the difficulties, and this is probably correct in outline.

First it should be emphasized that the Boltzmann



MAGNETIC FIELD LINES ON AN UNWRAPPED CYLINDER OR DOUGHNUT
FIGURE 6



MAGNETIC FIELD AND PLASMA MOTION IN THE PLANE PERPENDICULAR TO A CURRENT SHEET NEAR A NEUTRAL POINT
FIGURE 7

or Fokker-Planck equation is valid however seldom collisions occur (see Dungey 1958, Chapter II) and that by taking its zero and first order moments the usual equations are obtained except that the partial pressures take the form of symmetrical tensors. These tensors each have six independent components, and it is difficult to determine their values without a thorough treatment. One saving feature is that the pressure terms in the Ohm's law equation are likely to be small, and it is reasonable to take the approximation

$$\underset{\sim}{E} + \underset{\sim}{u} \times \underset{\sim}{B}/c = 0 \tag{4}$$

to be valid in most regions. Equation (4) implies that the magnetic field is "frozen in".

Secondly (4) puts such a great restriction on the flow, that its nature can virtually be deduced from (4) alone. If the flow were such as to greatly distort the magnetic field, the magnetic force would become dominant and prevent further stretching of the field lines. In fact, our problem can be posed as a steady state problem with given field and wind far from the earth. There is then an electrostatic potential ϕ, which from (4) is constant on a line of force. Given the magnetic field, ϕ is determined except on those field lines which have both feet on the ground. Where the behavior of ϕ is known, (4) determines the motion in relation to $\underset{\sim}{B}$. Thus the behavior in outline must be hydromagnetic.

A further difficulty is that the flow is probably

turbulent, particularly in the "wake" of the earth, and both Pioneer I and Explorer X observations show clear indications of some hydromagnetic form of turbulence. Even this, however, may not invalidate the use of (4) to give an outline, provided the turbulence does not lead to too much plasma slipping across the field. This topic is again one which is not well understood.

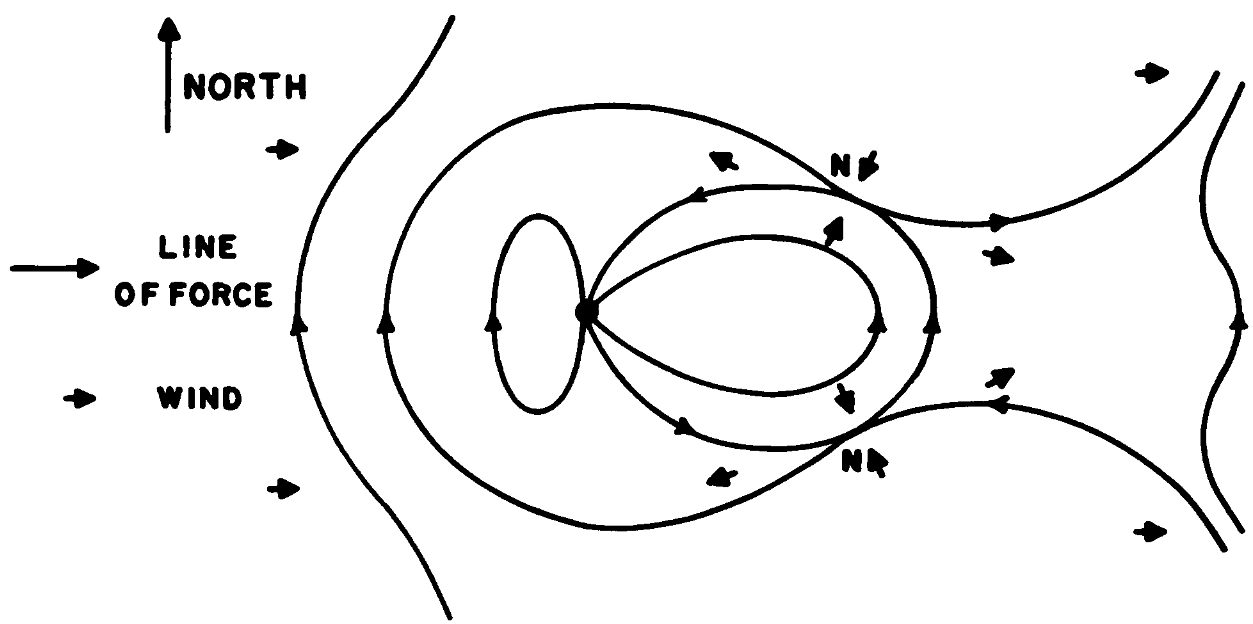
The flow has a special behavior near the neutral points, in which (4) is not valid. This has been discussed in detail before (Dungey 1958, § 6) and the conclusions will be quoted. Lenz's law, which states that the effects of induction "oppose the motion", fails and a large current density builds up in a sheet. This can be expressed in terms of the principal axes of $\partial B_i/\partial x_j$. Suppose two of the eigenvalues are positive, so that there is a surface covered by outward going lines. Then the other principal axis, corresponding to inward coming lines, makes a very small angle with this surface, after the current sheet has formed. The current sheet contains the part of the surface near the neutral point and the field reverses suddenly as the sheet is crossed.

In a plane perpendicular to the sheet and containing the incoming lines the field is shown in Fig. 7. The motion is also shown. Far enough from the neutral point (4) is valid and $\underset{\sim}{u} \times \underset{\sim}{B}$ corresponds to a uniform electric field out of the paper and the electric field at the neutral point has the same direction, though not satisfying (4). Near the

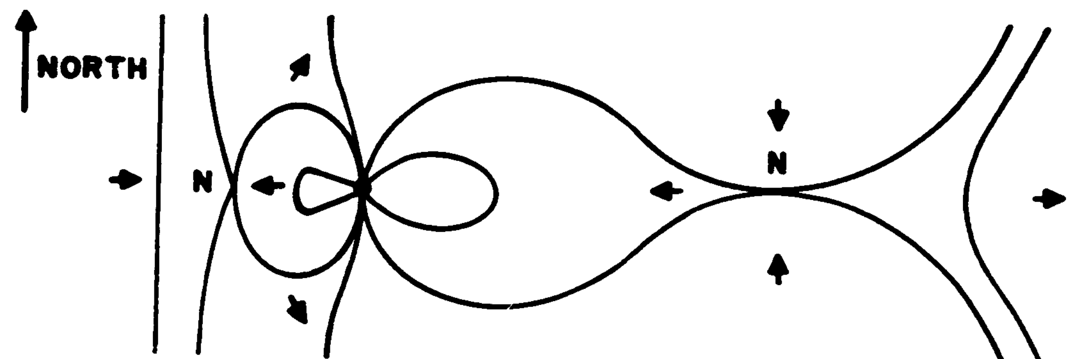
neutral point the electric field is required to drive the current.

The flow near a neutral point is driven by the magnetic force and, knowing that it has the form shown in Fig. 7, we are able to sketch the whole flow for given directions of interplanetary wind and field. For a northward field we obtain Fig. 8. The surface of lines of force joining the neutral points surrounds the earth as described in the previous Section. Outside this surface the flow is mainly in the direction of the interplanetary wind, but inside there are regions where the flow has the opposite direction; near the geomagnetic poles for instance. Inside the surface but off the plane of the diagram there is some flow in the direction of the wind. The behavior of the magnetic field could fit the observations from Explorer X, which seemed to pass near a neutral point in something like the right position; the observation would also be compatible with an interplanetary field parallel to the equatorial plane.

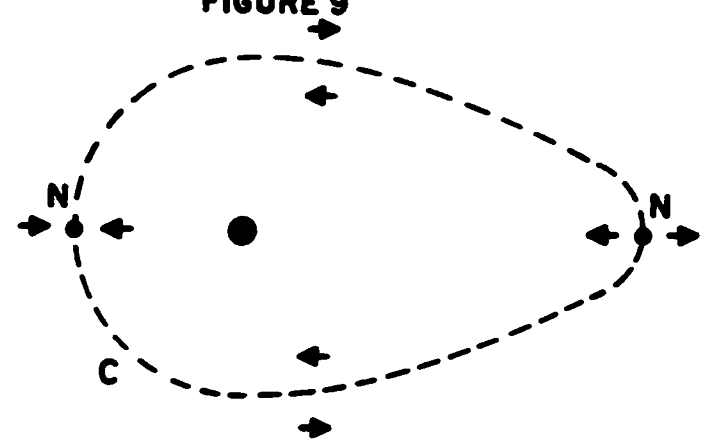
For a nearly southward field the flow must be as shown in Fig. 9. Lines of force connected to the neutral points cover surfaces topologically like a cylinder and a doughnut. Outside the doughnut the flow is mainly in the direction of the interplanetary wind. Inside the doughnut there are regions where it has the opposite direction, for instance near the neutral points. It is possible to obtain a plausible picture of the flow inside the doughnut, by assuming that the electric field is roughly uniform in the



MERIDIAN
PLANE MAGNETIC FIELD AND PLASMA MOTION FOR THE CASE OF A NORTHWARD INTERPLANETARY FIELD
FIGURE 8



MERIDIAN PLANE
MAGNETIC FIELD AND PLASMA MOTION FOR THE CASE OF A SOUTHWARD INTERPLANETARY FIELD
FIGURE 9



EQUATORIAL PLANE PLASMA FLOW FOR THE CASE OF A SOUTHWARD INTERPLANETARY FIELD
FIGURE 10

equatorial plane. It was pointed out in the last Section, that there are two lines of force which go direct from one neutral point to the other and these lie near the equatorial plane. In the equatorial plane, near these lines, is a closed curve C on which the component of $\underset{\sim}{B}$ perpendicular to the plane vanishes; inside C this component is northward and outside southward. Equation (4) then gives an idea of the flow, shown in Fig. 10, which shows a sudden reversal at the curve C. This reversal is due to the reversal of magnetic field, the electric field being roughly uniform. The flow inside the doughnut is determined by the flow in the equatorial plane, and it is now seen to be mainly in the opposite direction to the interplanetary wind. A general picture of the whole flow is thus obtained, but even on the hydromagnetic picture it is expected to be greatly complicated by turbulence.

4. Acceleration of Particles Near the Neutral Points

It has been shown previously that particles will gain energy from the electric field near the neutral point. It is suggested that some of these particles cause aurorae and with this in mind it is important to know the trajectories on which the energetic particles leave the accelerating region. The high current density in the discharge almost certainly causes "electrostatic" instabilities which result in plasma waves of about the plasma frequency travelling in the direction of the current. These waves will have a strong randomising effect on the velocities of the electrons, so that they come out in a wide cone of directions and possibly even in the direction of the electric field in spite of their

negative charge. Because of their greater mass the protons are much less affected by plasma waves and it is possible that their velocities are not much randomised. It is, therefore, important to study the trajectories of the protons in the accelerating region.

The current sheet is thin. If the current is carried by electrons with mean velocity v_e, the current density is $n\, e\, v_e$, and it is plausible that v_e corresponds to quite a large electron energy. As an idealisation we may suppose that the magnetic field varies linearly from B_1 to $-B_1$ as the current sheet is crossed, and take a thickness 2b for this change.

Then $$b \sim cB_1/4\pi\, n\, e\, v_e \qquad (5)$$

Consider now the particles coming into the current sheet from one side. The inflow is shown in Fig. 7 and described by (4). The "adiabatic invariant" $v_\perp^2/B$ is nearly constant until B becomes too small. The Larmor radius is $m\, c\, v_\perp/eB$ and B must not vary appreciably in this distance. Since the inflow is magnetically driven its energy density should be of the order of the magnetic energy density. The flow energy is concentrated in the protons and hence

$$B_1^{\,2} \sim 4\pi\, n\, m_p\, u^2 \qquad (6)$$

where u is the speed of inflow.

Putting u for $v_\perp$, the Lamor radius for a proton in the field B_1 may then be expressed as $cB_1/4\pi\, n\, e\, u$, which has the form of (5) with v_e replaced by u. Now v_e is expected to be much greater than the speed of inflow so

that the proton Larmor radius is bigger than b. This means that the incoming protons cross the current sheet, but they are subsequently turned round by the magnetic field and keep recrossing the sheet. Now protons vibrating across the sheet in this way exert a pressure and carry a current which thickens the sheet, and it may well be that this effect is sufficiently important to increase b to nearly the proton Larmor radius, so that an incoming proton no longer penetrates far beyond the sheet. The electron Larmor radius for velocity u is of course much smaller and the temperature of the incoming plasma is expected to be no more than $m_p u^2/2k$. Hence, it is likely that the Larmor radius of the electrons is a great deal less than b and therefore that the adiabatic approximation is good for electrons until they are so far inside the current sheet that B is much less than B_1; except that it may be spoilt by electrostatic instability waves. In any case the electrons would not exert the pressure across the sheet like the protons do. The electron pressure is probably nearly isotropic and smaller than this proton pressure. The next step, which is being undertaken, on this problem concerns the proton trajectories, assuming b to be about a proton Larmor radius and taking a smooth time-independent field. Such trajectories are being computed and may give a guide to the way in which the protons come out. Further study of the structure of the current sheet would also require these trajectories.

If particles accelerated in this wav are to cause aurorae they must be able to penetrate the ionosphere, so that their mirror points must not be too high. This is possible only for particles with very small values of the adiabatic invariant, and is a severe restriction. The fact that only a few particles penetrate might lead to narrow arc forms, but this has not been demonstrated.

It may be noted that Herlofson (1960) suggests injection of Van Allen particles at about 7 earth radii.

5. The Electric Field in the Ionosphere:SD

The plasma motion has been described in Section 3 on the basis of Eqn. (4) and using an electrostatic potential. The electric field is perpendicular to $\underset{\sim}{B}$ except near the neutral points, and as one follows a line of force in towards the earth the electrostatic potential ϕ remains constant until conduction effects become important. The electric field is important in the ionosphere at high latitudes and its variation with height can be adequately discussed taking the magnetic field to be vertical. If the z axis is vertical

$$j_x = -\sigma_1 \frac{\partial\phi}{\partial x} + \sigma_2 \frac{\partial\phi}{\partial y}$$

$$j_y = -\sigma_1 \frac{\partial\phi}{\partial y} - \sigma_2 \frac{\partial\phi}{\partial x}$$

$$j_z = -\sigma_0 \frac{\partial\phi}{\partial z}$$

and the vanishing of div $\underset{\sim}{j}$ requires

$$\sigma_1 \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + \frac{\partial}{\partial z} \sigma_0 \frac{\partial\phi}{\partial z} = 0 \qquad (7)$$

σ_0 and σ_1 being functions of z only.

Equation (7) could be integrated numerically, but approximations are sufficient to show that ϕ does not decrease very much as one comes down through the ionosphere as far as the E region. Suppose that

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \phi/b^2,$$

for our case $b \sim 1{,}000$ km

$$\sigma_0 \frac{\partial\phi}{\partial z} = - b^{-2} \int_0^z \sigma_1 \phi \, d z < - \frac{\phi}{b^2} \int_0^z \sigma_1 \, d z$$

Now $\int_0^z \sigma_1 d z$ never exceeds a value which is roughly given by the value of σ_0 in the E layer multiplied by a thickness of about 100 km. But σ_0 is proportional to the degree of ionization and so increases rapidly with height because of the air density falling off. Consequently above the E layer $\phi^{-1} \, \partial\phi/\partial z$ is less than $10^{-4} km^{-1}$ and ϕ is almost independent of height.

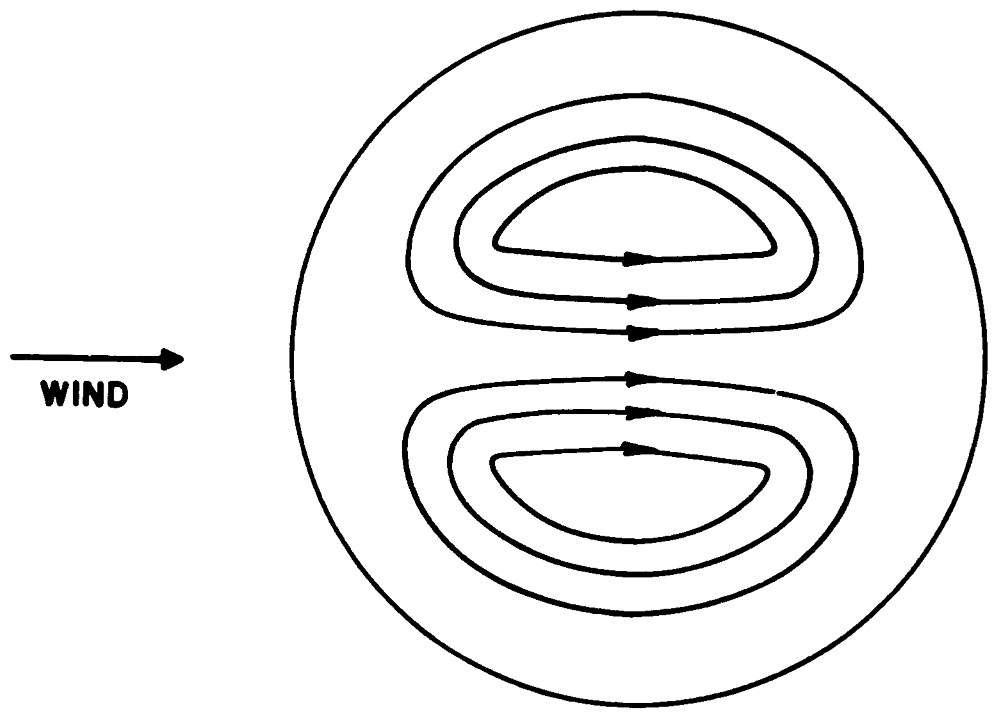
In the lower atmosphere the conductivity is negligible and no space charge is associated with the field, so that the potential satisfies Laplace's equation with the boundary condition $\phi = 0$ at the ground.

The Hall conductivity σ_2 gives rise to a current in the opposite direction to the plasma flow, and this is in fact the largest component of the integrated current, though it does not contribute to div $\underset{\sim}{j}$. The origin of this current can be understood by a physical argument. The electric field may be regarded as a motion of the magnetic field in accordance

with (4) and tries to make all charged particles drift with this velocity. On the other hand, collisions oppose any motion of the charged particles relative to the neutral air. High up the magnetic field wins and all charged particles move with the field while low down collisions dominate and all particles move with the air. The transition occurs for a particular kind of particle when its collision frequency passes the value of the gyrofrequency. Now the transition height for electrons is only 80 km. while for oxygen ions it is about 140 km. Thus there is an important region between in which the electrons move with the field while the ions move with the air. Consequently this region gives a substantial current in the opposite direction to $\underset{\sim}{v}$.

The motion of the field in the ionosphere is obtained by following the field lines in Fig. 9 and the result is shown in Fig. 11. The flow resembles the observed pattern known as SD, and a particularly striking feature is the very sudden reversal as one crosses from lines inside the doughnut to those which go off into space. This is observed near the auroral zone.

In addition to electric currents, the aurora shows up the SD pattern. Arcs tend to run along the flow lines, but this is unexplained. Irregularities in the arcs tend to move along in the direction of the motion of the magnetic field, and this motion is presumably just due to the drift of the primaries. The motion of irregularities detected by radio methods also illustrates the pattern; eastward before



DIRECTION OF MOTION OF THE FIELD
LOOKING DOWN ON THE POLAR REGION
FIGURE 11

midnight and westward after midnight at geomagnetic latitudes greater than 67°, and the other way round at lower latitudes. Again this can be explained simply by the motion of the plasma as a whole. There is thus substantial evidence for the existence of an electric field in the ionosphere with the morphology of Fig. 11.

The magnetic variations provide the most regularly recorded information on SD, and a systematic study of the IGY data is being undertaken by D. H. Fairfield. A point is worth noting here about the calculation of the currents from the magnetic variations. It is impossible to obtain the height distribution of the current even using all three magnetic components at the ground. On the other hand an approximation to the height-integrated horizontal components of the currents can be obtained using only the horizontal magnetic components. The procedure is simply to rotate the magnetic disturbance vector through a right angle and this would be the exact current, if it were at zero height. The point to be noted is that the horizontal current system so obtained is solenoidal and so has closed current lines. If $\underset{\sim}{b}$ is the horizontal magnetic disturbance and $\hat{\underset{\sim}{r}}$ is a unit vector, the result of rotating $\underset{\sim}{b}$ through a right angle is $\hat{\underset{\sim}{r}} \wedge \underset{\sim}{b}$. Now

$$\operatorname{div}\,(\hat{\underset{\sim}{r}} \wedge \underset{\sim}{b}) = - \hat{\underset{\sim}{r}} \cdot \operatorname{curl}\,\underset{\sim}{b}$$

because $\operatorname{curl}\,\hat{\underset{\sim}{r}} = \operatorname{curl}\,\operatorname{grad}\,r = 0.$

Thus the current lines obtained are closed and are expected to correspond to the Hall current, in which case the current lines are the equipotentials of ϕ. The

current σ_1 E runs across these equipotentials and so is not solenoidal and is also smaller than the Hall current. Even though the current patterns obtained by this procedure may differ appreciably from the true currents, the day to day changes in the patterns should be significant.

There appears to be a paradox in connection with any temporal change of the SD field. Any such change would propagate from space down towards the earth in the form of a hydromagnetic disturbance. The behavior of hydromagnetic waves when they reach the ionosphere has been investigated in connection with micropulsations, and it is found that the high conductivity of the earth is so important that the electric field in the ionosphere is very small for wave periods of a few seconds and even smaller for larger periods (Scientific Report No. 57). But earlier in this Section it was found that in the steady state the electric field extends right down to the E layer, and this should still be true under slowly varying conditions. It seems that the paradox can only be resolved by taking account of the geometry of the disturbance. The previous work was formulated in terms of the plane waves, and torsional waves must now be considered.

Suppose the geomagnetic field is vertical and use cylindrical coordinates r, ϕ, and z with their axis vertical. Let the disturbance high up be represented by a potential $\phi(r/a)^2$ for $r < a$, where $a \sim 1{,}000$ km. This will extend down to a height where conductivity is important, say 300 km, and we suppose that the electric field is initially small below

this height and is always zero at the ground. The plane wave theory indicates a magnetic field B_ϕ roughly proportional to r and independent of z. Taking the e.m.f. round the obvious circuit gives $B_\phi = 2\phi\ c\ r\ t/a^2\ H$. This gives $(\text{curl } \underset{\sim}{B})_z = 4\phi\ c\ t/a^2\ H$. Now the lower atmosphere has negligible conductivity, so that curl $\underset{\sim}{B}$ gives the displacement current, whence $E_z = 2\phi\ c^2 t^2/a^2\ H$. Thus a potential electric field is set up in the lower atmosphere to correspond with the potential in the ionosphere

If h is the height of the bottom of the ionosphere, ~ 100 km., the potential there grows as $2\phi\ h\ c^2 t^2/a^2\ H$, showing that the time scale for the growth of this potential $\sim (a^2 H/2c^2 h)^{1/2} \sim (\frac{3}{2})^{1/2} \frac{a}{c}$ which is a small fraction of a second. Thus potential electric fields do penetrate down the ionosphere, virtually instantaneously compared to the time scale of hydromagnetic phenomena. As a byproduct of this investigation it seems that the effect of these potential fields in the case .f micropulsations should be examined since, in the region where the conductivity is peculiar, they may produce unusual currents.

References

Dungey, 1958, Cosmic Electrodynamics (Cambridge University Press).

Dungey, 1961, Elliott, Explorer X and Others, see Report of Kyoto Conference, Proc. Jap. Phys. Soc., should be January 1962.

Herlofson, 1960, Phys. Rev. Letters 5, 414.